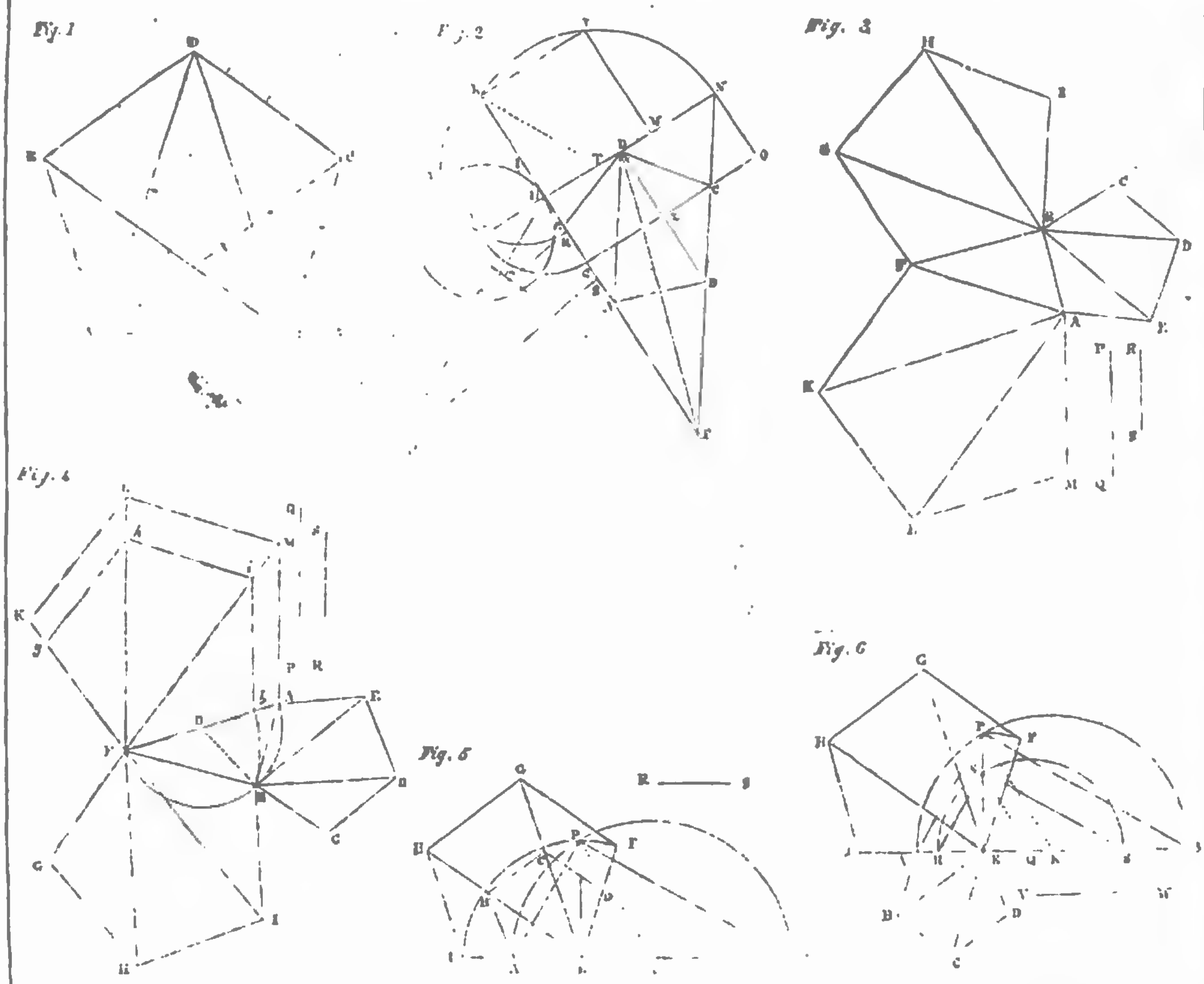
GEOMETRY OF THE PENTAGON.



THE GEOMETRY OF THE REGULAR PENTAGON.

Shewing the Method of its Construction, and its Conversion into an Equivalent Triangle, a Rectangular Parallelogram, a Square, and a Circle, together with the Method of Adding, Subtracting, Multiplying and Dividing regular Pentagons.

The regular pentagon or five-sided right lined figure, being of the greatest utility in the erection of various civil and military structures, it might reasonably be expected that its nature and properties would be perfectly understood by architects, engineers, and other practical individuals; and there can be no doubt but such is the case, in so far at least as its construction and mensuration are concerned; but with regard to its conversion into equivalent figures of different forms, and the other manipulations mentioned in the preamble, it may very justly be questioned if such a familiar knowledge of this very common and simple figure has yet been acquired, and the more especially so, as there is no system in our language in which those and similar operations are pointed out, or even hinted at.

Examples of the conversion of areas have, it is true, appeared in several of the late numbers of The Builder, but they are only to be regarded as particular cases of the general theory, and the attentive reader will readily perceive that the subject of the present paper is another application of the same principle, and one which is well adapted for displaying its efficiency, in the resolution of many curious and interesting problems, peculiar to architecture and several kindred branches of the constructive arts.

The delineation of the pentagon is effected in various ways, all of them simple and well-known; Euclid has given a very elegant and ingenious method of doing it, in the fourth book of his elements of geometry, and a much simpler method is pointed out by Ptolemy in the first book of his Almagest; but both these methods of construction, as well as the general process given by Renaldinus for all polygons, have reference to the inscription and circumscription of the figure in and about a circle; this, however, is not the object of the present inquiry, for it is not a given circle, but a given straight line, which is here supposed to constitute the datum on which the construction depends.

The methods of describing a pentagon upon a given straight line are also various, some of them very simple in practice and rigorous in their principles, while others are only approximative and difficult to apply. It is not, however, intended in this place to notice the several means of obtaining the same end: the main point is, to select from amongst many that method which is the easiest in practice, and the best adapted for general purposes.

The geometrical property, *that all the inward angles of every right-lined figure, when taken together, are equal to twice as many right angles, wanting four, as the figure has sides*, furnishes the easiest and most expeditious method of constructing the pentagon when the proper angular instruments are at hand, and the construction by this method will be correct in proportion to the delicacy of the instruments employed, and the skill and dexterity of the operator.

Since the pentagon has five sides, all the inward angles, or those contained between the adjacent sides of the figure, when taken two and two, will be expressed by the following theorem, viz:—

ten right angles — four right angles = six right angles;

and since a right angle contains 90 degrees, all the inward angles of the figure, five in number, will contain $90 \times 6 = 540$ degrees, being 108 degrees for each angle; for $540 \div 5 = 108$. This result distinctly indicates the method of constructing the pentagon, for we have only to make an angle of 108 degrees at each extremity of the given line, to obtain the directions of the two adjacent sides; then, by setting off in each of these directions a distance equal to the given line, the other two sides of the figure will from thence be readily determined.

This method, however, admits of considerable modification for practical purposes, and under such modification it is more readily applied in any particular case; the mode of construction is as follows:—

Let AB (fig. 1.) be the given straight line, on which it is proposed to construct a regular pentagon. At the extremity A of the given line AB, make the angles BAC, BAD and BAE, respectively equal to 36, 72, and 108 degrees, a process that can be performed with one application of the semicircular protractor. And, in like manner, at the extremity B of the given straight line AB, make the angles ABE, ABD, and ABC, respectively equal to 36, 72, and 108 degrees; then will C, D, and E, the points in which the several lines are found to intersect each other, be the angles of the pentagon. Draw the straight lines CD and DE, and ABCDE is the regular pentagon sought.

It would extend the present paper to too great a length to give a demonstration of the several constructions; it must therefore suffice to detail the steps of the operation in each case, and leave the demonstrations, either for exercise to the readers of the journal, or to their candour in taking it for granted, that the principles on which the operations depend are true. In the present instance, however, the truth of the construction is self-evident; and it is so